WESLEY COLLEGE
By daring & by doing

YEAR 12 MATHEMATICS SPECIALIST
SEMESTER ONE 2018
TEST 1: Complex Numbers

Name: _____

Wednesday 7 March

Time: 55 minutes **Mark** **/46=** **%**

- Answer all questions neatly in the spaces provided. **Show all working.**
- You are permitted to use the Formula Sheet in **both** sections of the test.
- You are permitted one A4 page (one side) of notes in the calculator assumed section.

Calculator free section **Suggested time: 25 minutes** **/22**

Question 1 (6 marks)

$$f(z) = z^3 - 5z^2 + 17z - 13$$

a) Show $(z-1)$ is a factor of $f(z)$

[1]

b) Re-write $f(z) = (z-1)Q(z) + R$

[2]

c) Hence find all the roots of the equation, giving your answers in the form $a + ib$ where a and b are integers.

[3]

Question 2 (9 marks)

The complex numbers z_1 z_2 and z_3 are given by

$$z_1 = 7 - i \qquad z_2 = 1 + i\sqrt{3} \qquad z_3 = a + ib$$

where a and b are real constants.

a) Given $|z_1 z_3| = 50$, find $|z_3|$

[2]

b) Given also $\arg\left(\frac{z_2}{z_3}\right) = \frac{7\pi}{12}$, find $\arg(z_3)$

[2]

c) Determine the values of a and b.

[2]

d) Show that $\frac{z_1}{z_3} = \frac{1}{5}(4 + 3i)$

[3]

Question 3 (3 marks)

Given $w = a + ib$, find the values of a and b when $2w - 3\overline{w} = 3 - 20i$.

[3]

Question 4 (4 marks)

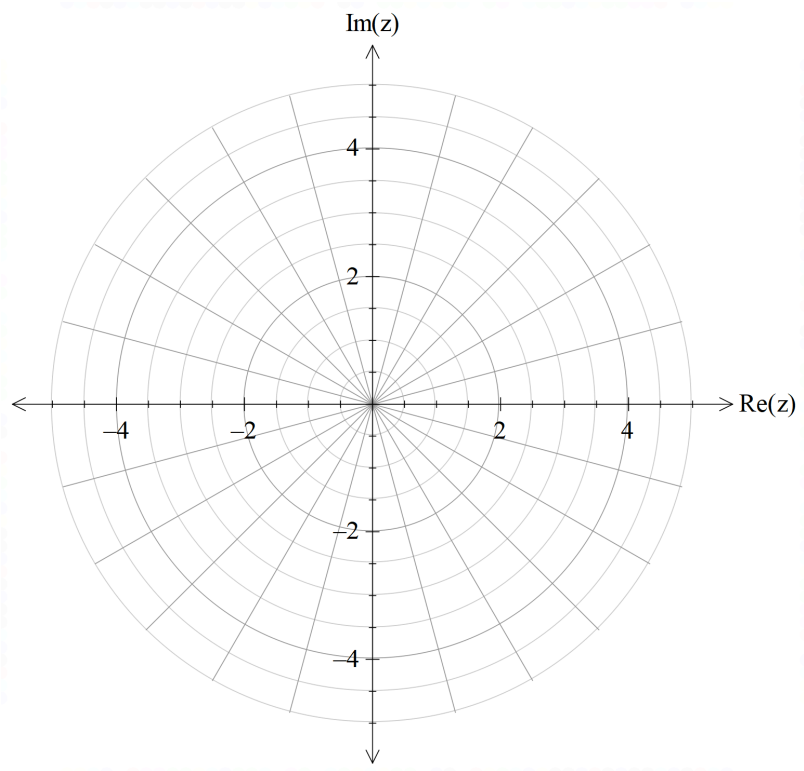
If $z = 2cis\left(\frac{\pi}{6}\right)$, illustrate, and label, on an Argand diagram the points:

i. z

ii. $\overline{z}$

iii. $\left(\overline{z}\right)^2$

iv. $\frac{\left(\overline{z}\right)^2}{|z|}$



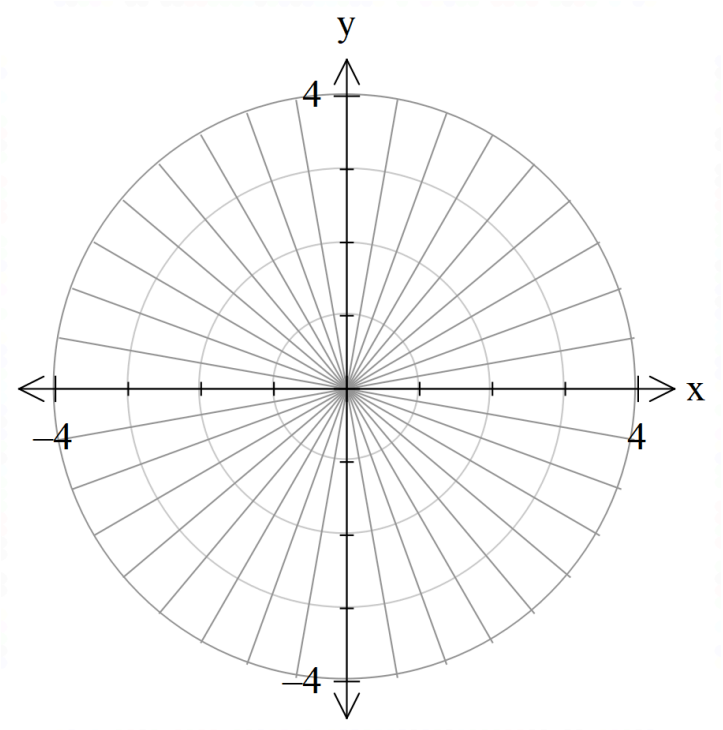
[4]

Name: _____

Calculator assumed section **Suggested time: 25 minutes** **/24**

Question 5 (5 marks)

Illustrate the solutions to the equation $z^3 = -4 + 4\sqrt{3}i$.



[6]

Question 6 (5 marks)

The complex number z is given by $z = \dfrac{1+i\sqrt{3}}{1-i\sqrt{3}}$.

a) State z in the form $\lambda\left(1-i\sqrt{3}\right)$, where λ is rational number you should find.

[1]

b) State the modulus and argument of z.

[2]

c) Hence, or otherwise, find the modulus and argument of $\left(\dfrac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)^4$

[2]

Question 7 (5 marks)

Given that $\left|z+1-4i\right| = 1$

a) sketch, in an Argand diagram, the locus of z.

[2]

b) find the maximum value of arg(z) in degrees to one decimal place.

[3]

Question 8 (9 marks)

The complex number $w = x + iy$, where x and y are real, satisfies the equation

$$|w + 1 + 8i| = 3|w + 1|.$$

The complex number w is represented by the point P in the Argand diagram.

a) Show that the locus of P is a circle and state the centre and radius of this circle.

[3]

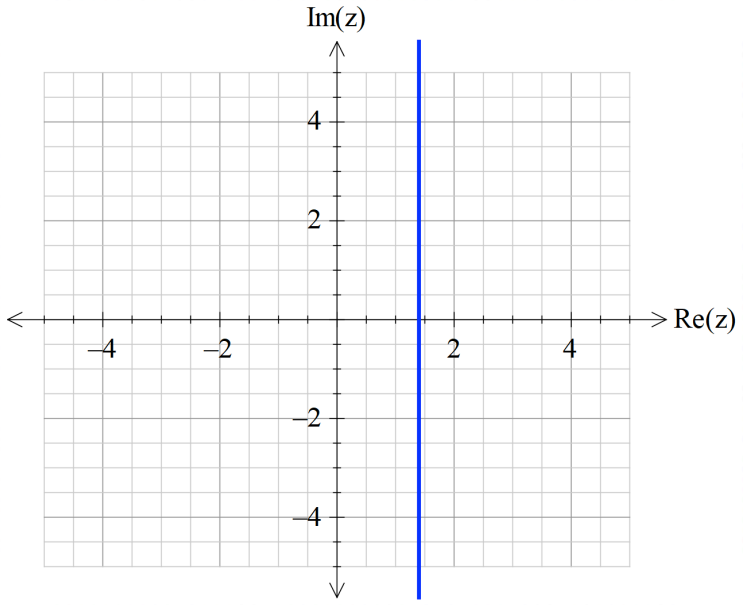
Question contd on next page ...

b) The locus $|w| = \left|w - \frac{14}{5}\right|$ is represented on the Argand diagram below.

i. Explain why the locus is as shown.

[2]

ii. Add the locus of P to the diagram.



[2]

c) Find the complex numbers corresponding to the points of intersection of these loci, giving your answers in the form $a + ib, \quad a, b \in$.

[2]